Exponential Growth, Decay, and Compound Interest
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Math 151 - 901

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9 Compound Interest

In this chapter we will learn about compound interest.

9.1 Introduction

Recall: when we want to compute a percentage of a number, we multiply the number by the percentage in **decimal form.** For example, 15% of 20 is $(0.15)(20) = 3$. To convert from percent to decimal, we divide by 100.

Let x be a number. If we say x has increased by 4%, then what is the new value? We convert 4% to decimal: 0.04. Then we add 4% of x to x:

$$x + 0.04x.$$

We can combine these terms, though, and write $(1 + 0.04)x = 1.04x$.

Example – Exponential Growth

1. Sandy's salary increases by 15% from \$15,000. What is her current salary?

   Here our *rate* is 15%, which we convert to 0.15. Our principal, or starting value, is \$15,000. To get the new value, we compute

   $$\$15,000 + (0.15)(\$15,000) = (1.15)(\$15,000) = \$17,250.$$

   Notice here we kept track of our units. Our final answer is in **dollars.**

2. Suppose Jane borrows \$3,000 at an interest rate of 3% compounded yearly. Assume no payments are made on the loan.

   (a) Find the amount owed after 1 year.

   Here again we have a rate of 3%, or in decimal 0.03. The interest is compounded yearly, and we are asked to find the amount owed after 1 year. Our principal is \$3,000. Then $(1.03)(\$3000) = \$3090$.

   (b) Find the amount owed after 2 years.

   To find the amount owed after 2 years, we must take the amount owed after 1 year and repeat the computation. Notice that we don't just add \$90. That is, the answer **is not** $\$3000 + (0.03)(\$3000) + (0.03)(\$3000)$.

   We compute:

$$(1.03)(\$3090) = \$3182.70$$

   (c) Write a general formula for the amount Jane owes after t years, assuming no payments are made.

   Notice, with each passing year, we take the last years value and multiply by 1.03 to compute a 3% increase. In (a) we saw after one year she owed $(1.03)(\$3000)$. After two years she owed $(1.03)(\$3090)$, or more generally, $(1.03)[(1.03)(\$3000)] = (1.03)^2(\$3000)$. In general, after t years Jane will owe $(1.03)^t(\$3000)$.

9.2 Compound Interest Formula - Discrete

To calculate the final amount in a compound interest problem where things grow discretely (the interest accrues a defined number of times per year), we use the following formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

- A is the final amount.
- P is the initial amount or initial principal.
- r is the interest rate in decimal form. For example, 15% is 0.15, or 4% is 0.04.
- n is the number of times interest is applied per time period.
- t is the number of time periods elapsed.

For our purposes, t is the **number of years** and n is the **number of times interest is applied per year.** We have special keywords for n. For example

- Compounded

- yearly: $n = 1$
- semiannually: $n = 2$.
- quarterly: $n = 4$
- monthly: $n = 12$
- daily: $n = 365$

In compound interest word problems, one is tasked with identifying **given values** and substituting the given values into the formula properly.

Example – Compound Interest

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