Directions

You should not consider this packet to be anything other than practice for the Midterm. There may be questions on the Midterm that do not resemble these questions entirely. Review all your notes, previous quizzes, and homework. In doing so there should be no surprises. This may not be an exhaustive study list, but I thought I'd include one to get you started.

1. Chapter 1: Voting Methods
   - What is a preference ballot? What is a preference schedule?
   - Plurality method – how to apply, pros and cons
   - Borda count method – how to apply, pros and cons
   - Plurality-with-elimination method – how to apply, pros and cons
   - Pairwise comparison method – how to apply, pros and cons
   - What is a Condorcet candidate? How do search for a Condorcet candidate?
   - What are the four major fairness criteria?
   - What is Arrow's Impossibility Theorem?
2. Chapter 3: Mathematics of Sharing
   - Fair Value Systems: What is the concept of fair share? Know how to calculate prices of fair shares.
   - Divider-chooser – How to tell who divides? Is it better to be divider or chooser?
   - Lone-divider – Process of assigning shares.
   - Sealed Bids – How to calculate fair shares? What is a surplus? How is the surplus calculated/distributed?
   - Method of markers – What are fair shares to the players? Know the process to distribute items.
3. Chapter 9: Population Growth Models
   - Arithmetic (linear) Sequences: What is the common difference? How do you find terms? How do you find which term a specific number is in the sequence? How do you sum the first N terms?
   - Geometric (exponential) Sequences: What is the common ratio? How do you find terms? How do you sum the first N terms?
   - Word problems: Recognize when to use arithmetic or geometric sequences. Be able to figure out common ratio or difference.
   - How to tell a sequence is linear or exponential.

Chapter 1: Voting Methods

Consider the following preference schedule:

Number of Voters	29	21	18	10	1
1st Choice	D (116)	A (84)	B (72)	C (40)	C (4)
2st Choice	C (87)	C (63)	A (54)	B (30)	B (3)
3st Choice	A (58)	B (42)	C (36)	A (20)	D (2)
4st Choice	B (29)	D (21)	D (18)	D (10)	A (1)

1. Find the winner using the plurality method.

Solution

We need to count the total first-place votes for each candidate:

$$\begin{aligned} A &= 21 \\ B &= 18 \\ C &= 11 \\ D &= 29 \end{aligned}$$

Since D has the most first-place votes, they win using the plurality method.

2. Find the winner using the Borda count method.

Solution

See the table for the point values. Remember, Borda awards candidates points for each column, depending on their position in that column. The number of points for a rank in a column (1st, 2nd, 3rd, 4th) times the number of voters is the number of points for that column. We have

$$\begin{aligned} A &= 58 + 84 + 54 + 20 + 1 = 217 \\ B &= 29 + 42 + 72 + 30 + 3 = 176 \\ C &= 87 + 63 + 36 + 40 + 4 = 230 \\ D &= 116 + 21 + 18 + 10 + 2 = 167 \end{aligned}$$

C has the most points, so they are the winner using Borda count.

3. Find the winner using the plurality-with-elimination method.

Solution

There are a total of 79 voters in this election. In order to have a majority, a candidate needs to have more than $79/2 = 39.5$ votes, i.e. 40 or more votes for a majority. We go ahead with the method:

Round 1	Round 2	Round 3
A: 21	~~A~~: 21	~~A~~: 21
B: 18	B: $18 + 11 = 29$	B: $18 + 11 + 21 = 50$
~~C~~: 11	~~C~~: 11	~~C~~: 11
D: 29	D: 29	D: 29

Round 1: No majority candidate. C has the least votes and is eliminated, their 11 votes go to B according to the preference schedule. Round 2: No majority candidate. A has the least votes and is eliminated, their 21 votes go to B according to the preference schedule. Round 3: B wins with a majority of the votes (50 votes.)

4. Find the winner using the pairwise comparisons method.

Solution

We have four candidates, so there should be $\frac{(4)(3)}{2} = 6$ pairs to compare. For each pair, we look to see the total number of votes on candidate has over the other, and vice versa.

A	B
29	18
21	10
	1
50	28

A	C
21	29
18	10
	1
39	40

A	D
21	29
18	1
10	
49	30

B	C
18	29
	21
	10
	1
18	61

B	D
21	29
18	
10	
1	
50	29

C	D
21	29
18	
10	
1	
50	29

So our point totals are A: 2, B: 1, C: 3, D: 0.

5. Use this election to show why the plurality-with-elimination violates the Condorcet criterion.

Solution

Using our work from problem 4. we can see that while C is a Condorcet candidate, the winner for this election under plurality with elimination is B. Therefore plurality-with-elimination violates the Condorcet criterion.

Chapter 3: Mathematics of Sharing

Fair Value and Lone-Divider

Alice, Bob, and Carlos are dividing among themselves the family farm equally owned by the three of them. The farm was divided into three shares, namely s_1, s_2, and s_3. Consider the table below illustrating how each of them values the three shares:

	s_1	s_2	s_3
Alice	38%	28%	34%
Bob	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$
Carlos	34%	40%	26%

1. Who was the divider? How do you know?

Solution

Bob was the divider, since all the shares are of equal value to him.

2. Write down the bid lists for each player. (What each player considers to be a fair share.)

Solution

Alice: $\{s_1, s_3\}$, Bob: $\{s_1, s_2, s_3\}$, and Carlos: $\{s_1, s_2\}$.

3. Find all possible fair **divisions** (so, assingments) of the assets.

Solution

The trick here is to see what happens when Bob gets s_1, s_2, or s_3.

Player	Alice	Bob	Carlos
Assignment 1	s_1	s_3	s_2
Assignment 2	s_3	s_2	s_1
Assignment 3	s_3	s_1	s_2

4. Of the fair divisions found, which one is the best?

Solution

Assignment 1 above is the best, since it is envy-free. In other words, each player received their most valued share.

Sealed Bids

Ana, Belle, and Chloe are dividing four pieces of furniture using the method of sealed bids.

Steps:	**Ana**	**Belle**	**Chloe**
Bids (made in private, sealed envelope)			
Dresser	$150	$300	$275
Desk	$180	$150	$165
Vanity	$170	$200	$260
Tapestry	$400	$250	$500
Total of Bids	**$900**	**$900**	**$1200**
Fair Share	**$300**	**$300**	**$400**
Initial Allocation of Items	**Desk**	**Dresser**	**Vanity Tapestry**
Value of Items Won	**$180**	**$300**	**$760**
Estate Payment/Receipt	**Receives $120**	**Nothing**	**Pays $360**
Estate Surplus Share	**$80**	**$80**	**$80**
Final Allocation (Items and Net Payment/Receipt)	**Desk, Receives $200**	**Dresser, Receives $80**	**Vanity, Tapestry, Pays $280**

Estate Work Sheet

This Work sheet is used with the Sealed Bids Estate Settlement work sheet to calculate the amount of surplus each player received from the Estate.

A. Sum of all monies paid into the Estate A. $360

B. Sum of monies paid out of the Estate B. $120

C. The difference of A and B = total surplus C. $240

D. Each player's share if total surplus D. $80

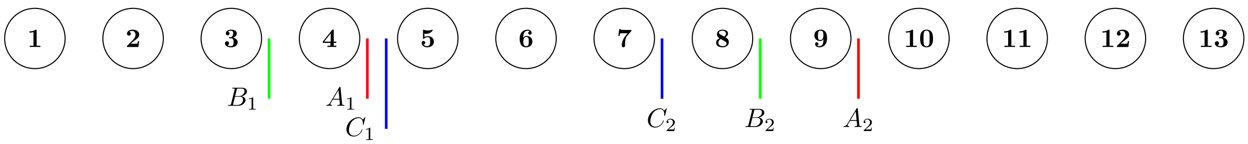
The Method Of Markers

Allison (A), Beth (B), and Carl (C) are dividing 13 cars and decide to use the method of markers.

A: immediately to the right of cars 4 and 9

B: immediately to the right of cars 3 and 8

C: immediately to the right of cars 4 and 7



1. Which segments does each player consider a fair share? (List for each player.)

Solution

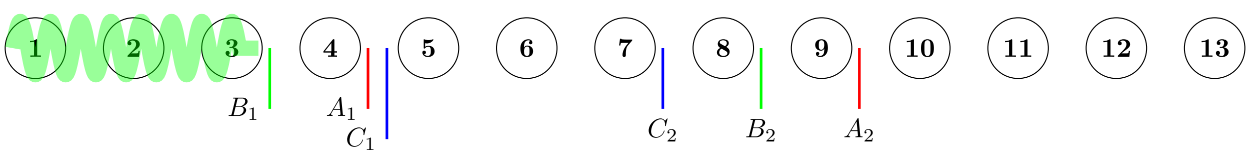
Putting the markers down, we can see that they consider the following segments to be fair shares:

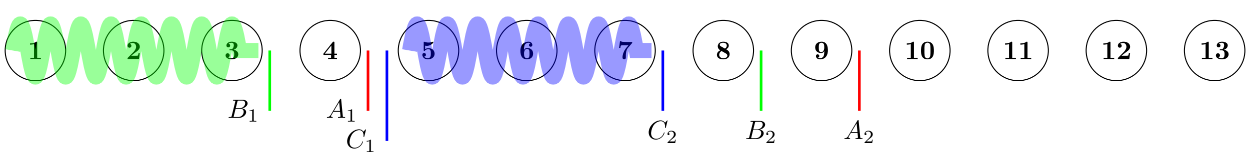
A: 1-4, 5-9, 10-13

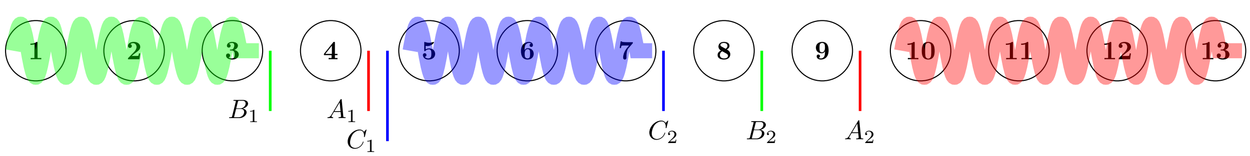
B: 1-3, 4-8, 9-13

C: 1-4, 5-7, 8-13

2. Describe the allocation of items to each player.







Solution

B gets 1-3, C gets 5-7, and A gets 10-13.

3. Which items are left over?

Solution

Items 4, 8, and 9 are left over.

Chapter 9: Population Growth Models

1. Consider the sequence $5, 9, 13, 17, \ldots$.

   (a) Is the sequence linear or exponential? Find the common difference or common ratio.

   Solution

   The sequence is linear. Computing the differences we have

   $$\begin{aligned} 9 - 5 &= 4 \\ 13 - 9 &= 4 \\ 17 - 13 &= 4 \end{aligned}$$

   and hence there is a *common* difference of 4.

   (b) Find T_4.

   Solution

   Since we have T_3 we can use the recursive formula:

   $$T_4 = T_3 + d = 17 + 4 = 21.$$

   (c) Find T_{17}.

   Solution

   We should use the explicit formula here:

   $$T_{17} = 5 + (17)(4) = 5 + 68 = 73.$$

   (d) What is the 38th term of this sequence?

   Solution

   We should use the explicit formula to find the 38th term, or T_{37}, of the sequence:

   $$T_{37} = 5 + (37)(4) = 153.$$

   (e) The number 185 is what term of the sequence?

   Solution

   We can set up an equation using the explicit formula. Remember, this question is asking "for which N is $T_N = 185$?" So we have

$$\begin{aligned} 185 &= 5 + 4N \\ 180 &= 4N && \text{(Subtract 5)} \\ 45 &= N && \text{(Divide by 4)} \end{aligned}$$

So 185 is the 46th term, or T_{45}.

(f) The number 233 is what term of the sequence?

Solution

Following what we did in part (e) we have

$$\begin{aligned} 233 &= 5 + 4N \\ 228 &= 4N && \text{(Subtract 5)} \\ 57 &= N && \text{(Divide by 4)} \end{aligned}$$

So 233 is the 58th term or T_{57}.

(g) What is $\displaystyle\sum_{i=0}^{15} T_i$?

Solution

Using the sum formula we have

$$\sum_{i=0}^{15} = \frac{(T_0 + T_{15})(16)}{2}$$

and using the explicit formula to find T_{15} we have

$$T_{15} = 5 + (15)(4) = 65.$$

Plugging everything in and computing:

$$\sum_{i=0}^{15} T_i = \frac{(5 + 65)(16)}{2} = 560$$

2. Consider the sequence $34, 28, 22, 16, \ldots$.

(a) Is the sequence linear or exponential? Find the common difference or common ratio.

Solution

The sequence is linear. Computing the differences we have

$$28 - 34 = -6$$
$$22 - 28 = -6$$
$$16 - 11 = -6$$

and hence there is a *common* difference of -6.

(b) Find T_4.

Solution

Since we have T_3 we can use the recursive formula:

$$T_4 = T_3 + d = 16 + (-6) = 10.$$

(c) Find T_{19}.

Solution

We should use the explicit formula here:

$$T_{19} = 34 + (19)(-6) = 34 - 114 = -80.$$

(d) What is the 33rd term of this sequence?

Solution

We should use the explicit formula to find the 33rd term, or T_{32}, of the sequence:

$$T_{32} = 34 + (34)(-6) = -170.$$

(e) The number -92 is what term of the sequence?

Solution

We can set up an equation using the explicit formula. Remember, this question is asking "for which N is $T_N = -92$?" So we have

$$\begin{aligned} -92 &= 34 - 6N \\ -126 &= -6N && \text{(Subtract 34)} \\ 21 &= N && \text{(Divide by } -6\text{)} \end{aligned}$$

So -92 is the 21st term, or T_{20}.

(f) The number -128 is what term of the sequence?

Solution

We can set up an equation using the explicit formula. Remember, this question is asking "for which N is $T_N = -92$?" So we have

$$\begin{aligned} -128 &= 34 - 6N \\ -162 &= -6N && \text{(Subtract 34)} \\ 27 &= N && \text{(Divide by } -6\text{)} \end{aligned}$$

So -128 is the 27th term, or T_{26}.

(g) What is $\sum_{i=0}^{15} T_i$?

Solution

Using the sum formula we have

$$\sum_{i=0}^{15} = \frac{(T_0 + T_{15})(16)}{2}$$

and using the explicit formula to find T_{15} we have

$$T_{15} = 34 + (15)(-6) = -56.$$

Plugging everything in and computing:

$$\sum_{i=0}^{15} T_i = \frac{(34 - 56)(16)}{2} = -704$$

3. Consider the sequence $2200, 1760, 1408, 1126.4\dots$.

(a) Is the sequence linear or exponential? Find the common difference or common ratio.

Solution

The sequence is exponential. If you try to find a common difference you will fail. Looking for a common ratio:

$$\frac{T_1}{T_0} = \frac{1760}{2200} = 0.8$$

$$\frac{T_2}{T_1} = \frac{1408}{1760} = 0.8$$

$$\frac{T_3}{T_2} = \frac{1126.4}{1408} = 0.8$$

So the *common* ratio is 0.8.

(b) Find T_4.

Solution

Since we have T_3 we should use the recursive formula, so:

$$T_4 = T_3(r) = 1126.4(0.8) = 901.12$$

(c) Find T_{11}.

Solution

We should use the explicit formula:

$$T_{11} = T_0(r^N) = 2200(0.8^{11}) = 188.9786$$

(d) What is the 7th term of this sequence?

Solution

The 7th term, or T_6, of the sequence can be found with the explicit formula:

$$T_6 = 2200(0.8^6) = 576.7168.$$

(e) What is $\sum_{i=0}^{13} T_i$?

Solution

Using the sum formula we have

$$\sum_{i=0}^{13} T_i = \frac{T_0(r^N - 1)}{r - 1} = \frac{2200(0.8^{14} - 1)}{0.8 - 1} = \frac{2200(0.8^{14} - 1)}{-0.2} \approx 10516.2149$$

4. Consider the sequence $20, 28, 39.2, 54.88\ldots$.

(a) Is the sequence linear or exponential? Find the common difference or common ratio.

Solution

The sequence is exponential. If you try to find a common difference you will fail. Looking for a common ratio:

$$\frac{T_1}{T_0} = \frac{28}{20} = 1.4$$

$$\frac{T_2}{T_1} = \frac{39.2}{28} = 1.4$$

$$\frac{T_3}{T_2} = \frac{54.88}{39.2} = 1.4$$

So the *common* ratio is 1.4.

(b) Find T_4.

Solution

Since we have T_3 we should use the recursive formula, so:

$$T_4 = T_3(r) = (54.88)(1.4) = 76.832$$

(c) Find T_9.

Solution

We should use the explicit formula:

$$T_9 = T_0(r^N) = 20(1.4^9) \approx 413.2209$$

(d) What is the 7th term of this sequence?

Solution

The 7th term, or T_6, of the sequence can be found with the explicit formula:

$$T_6 = 20(1.4^6) \approx 150.5907$$

(e) What is the sum of the first 15 terms of the sequence? Use sigma notation and the appropriate sum formula.

Solution

In sum notation, the sum of the first 15 terms is written (along with the appropriate formula)

$$\sum_{i=0}^{14} T_i = \frac{T_0(r^{15}-1)}{r-1}.$$

Plugging in we have

$$\sum_{i=0}^{14} T_i = \frac{20(1.4^{15}-1)}{1.4-1} = \frac{20(1.4^{15}-1)}{0.4} \approx 7728.4048.$$

5. Some conceptual stuff.

(a) A population **increases** exponentially by 12%. What is the common ratio r?

Solution

The common ratio r of a population that increases exponentially by 12% is $1 + 0.12 = 1.12$.

(b) A population **increases** exponentially by 13.5%. What is the common ratio r?

Solution

The common ratio r of a population that increases exponentially by 13.5% is $1+0.135 = 1.135$.

(c) A population **decreases** exponentially by 19%. What is the common ratio r?

Solution

The common ratio r of a population that decreases exponentially by 19% is $1 - 0.19 = 0.81$.

(d) A population **decreases** exponentially by 9.5%. What is the common ratio r?

Solution

The common ratio r of a population that decreases exponentially by 9.5% is $1-0.095 = 0.905$.

(e) Why can't a linear sequence be an exponential sequence, or vice versa?

Solution

A linear sequence cannot be an exponential sequence since it increases or decreases by a **constant** value, or common difference, whereas an exponential sequences increases (or decreases) by a value that grows, as the name suggests, exponentially.